

Inclusive jet production in electron-nucleon collisions

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Based on [G. Abelo, R. Boughezal, X. Liu, FP
PLB \(2016\) 52-59 \[arXiv:1607.04921\]](#)



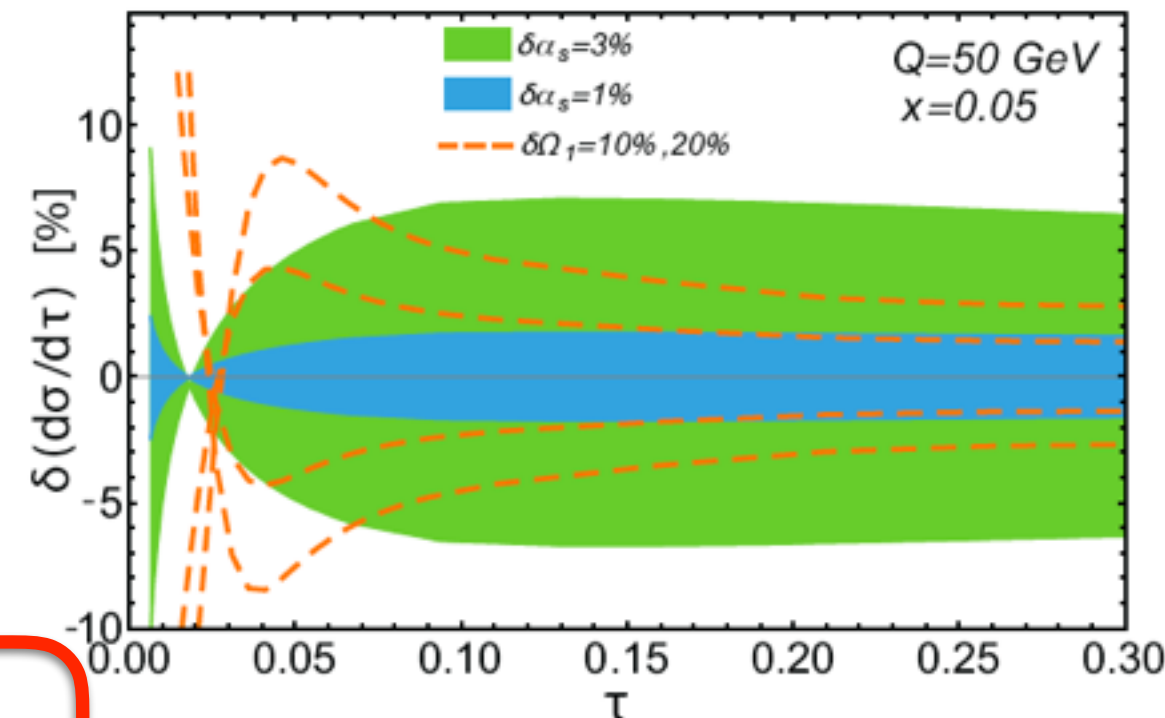
NORTHWESTERN
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POETIC 2016
November 16, 2016



Jet physics at the EIC

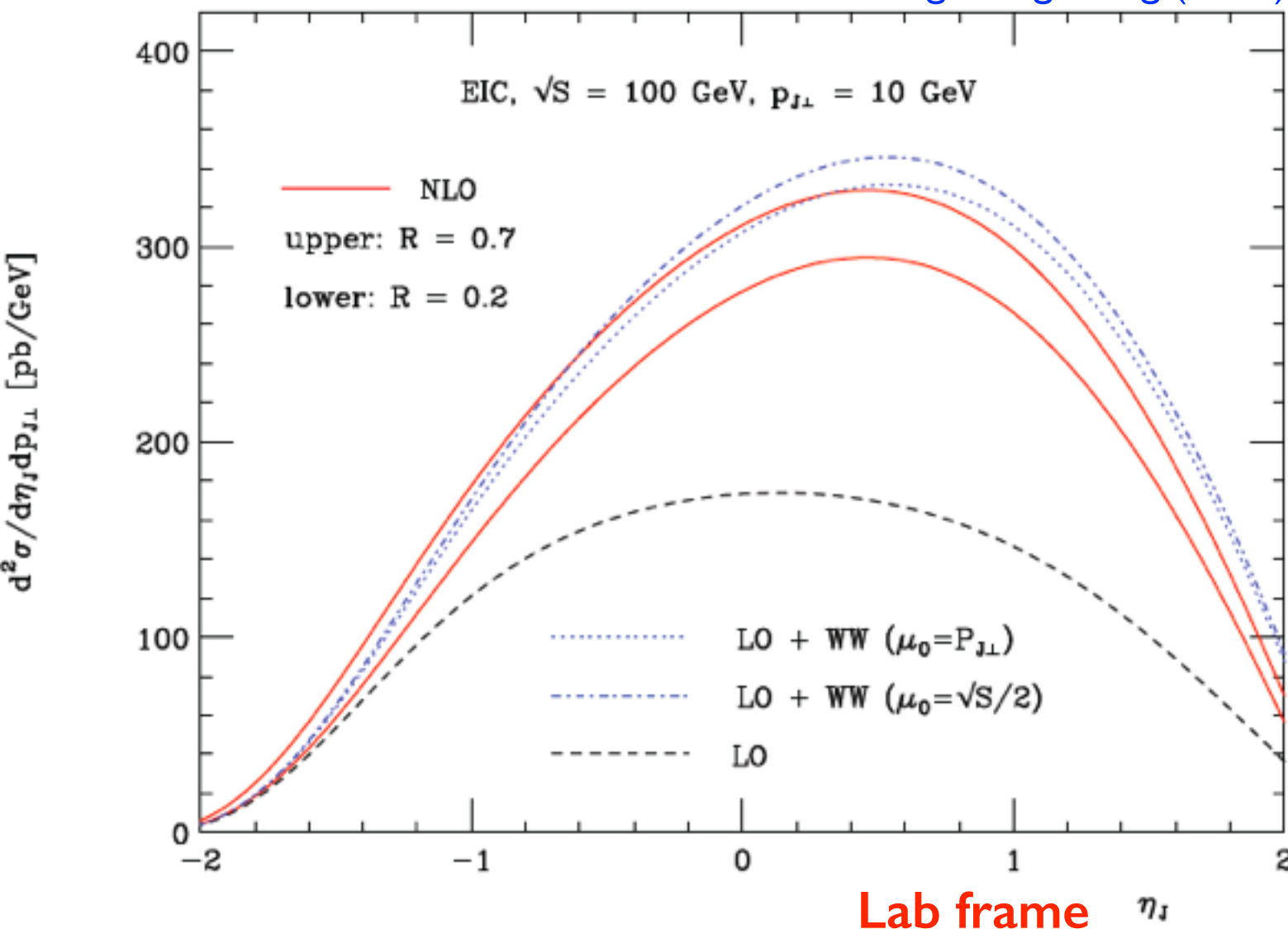
- Numerous physics motivations for studying jet production at a future EIC
- Measurement of the strong coupling constant [D. Kang, Lee, Stewart \(2013\)](#)
- Determination of higher-twist properties of the proton [Z. Kang, Metz, Qiu, Zhou \(2011\)](#)
- Determination of parton distribution functions
- Measure properties of the nuclear medium with event shapes [Z. Kang, Liu, Mantry, Qiu \(2012\)](#)



The precision of an EIC plays a critical role in all of these measurements!

The challenge: large corrections

Hinderer, Schlegel, Vogelsang (2015)



- Large NLO perturbative corrections, $O(100\%)$
- Important, but not dominant, corrections from photon-initiated processes
- Does the perturbative series converge at NNLO?
- Are the NNLO corrections dominated by a single channel?

Goals:

- Investigate the NNLO corrections to EIC jet production for its intrinsic interest
- Show that techniques for LHC calculations can also enable precision EIC studies

Definition of the process

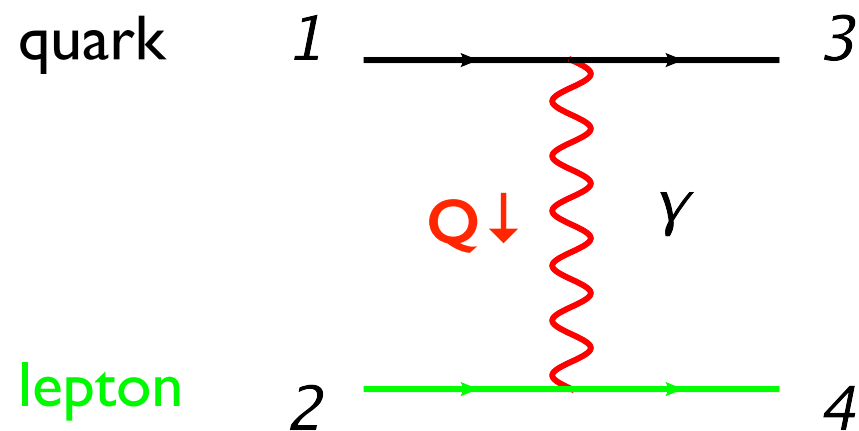
DIS: $eN \rightarrow eN$

- lepton tagged
- Cut on Q^2
- hard scale: Q

Inclusive jet production: $eN \rightarrow jX$

- lepton *not* tagged
- Cut on p_{Tjet}
- hard scale: p_{Tjet}

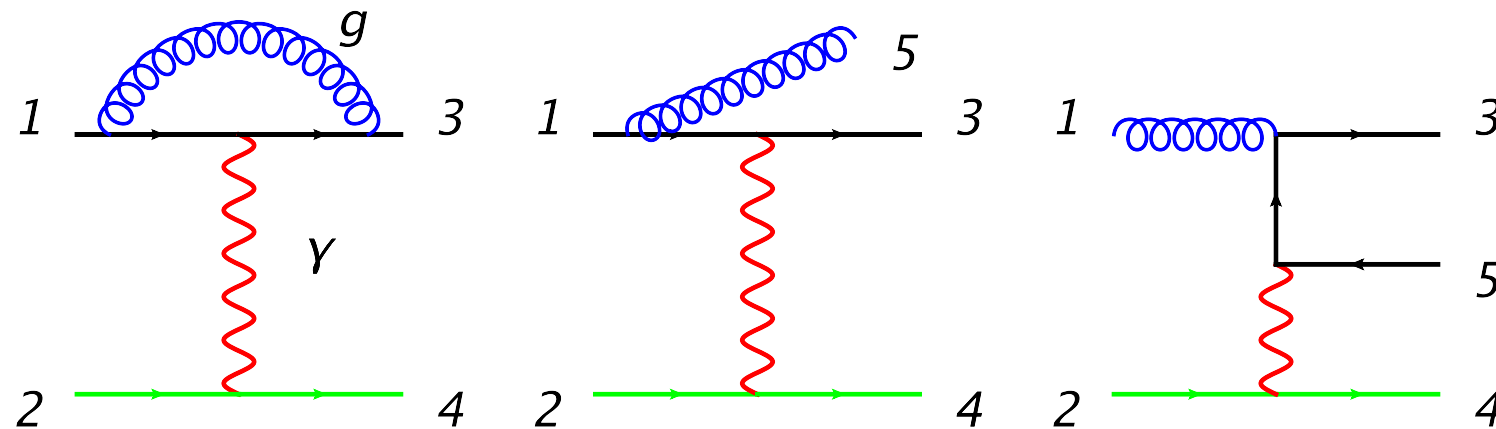
$$q(p_1) + l(p_2) \rightarrow q(p_3) + l(p_4)$$



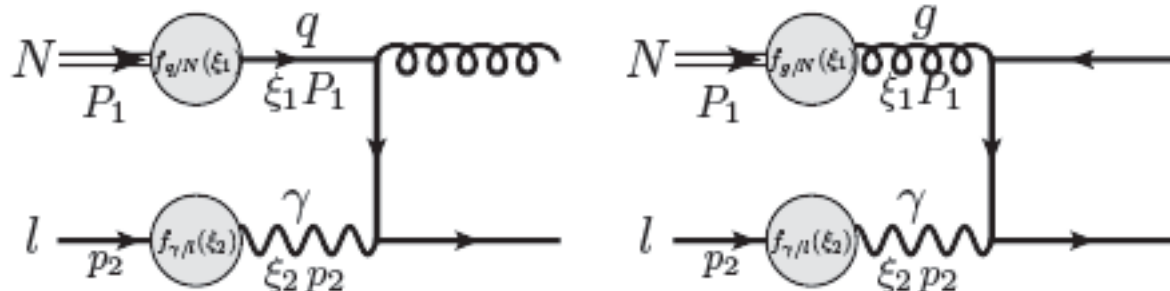
- **Leading order:** identical for both processes, lepton recoils against a jet

NLO $\mathcal{O}(\alpha^2\alpha_s)$ corrections

- Typical real and virtual corrections to the quark-lepton scattering processes; new contribution from gluon-lepton scattering \rightarrow calculation amenable to standard techniques



- **New configuration:** lepton collinear to the beam ($Q^2 \sim 0$), with two jets balancing in the transverse plane; on-shell photon scattering with quark \rightarrow differentiates DIS and inclusive jet production

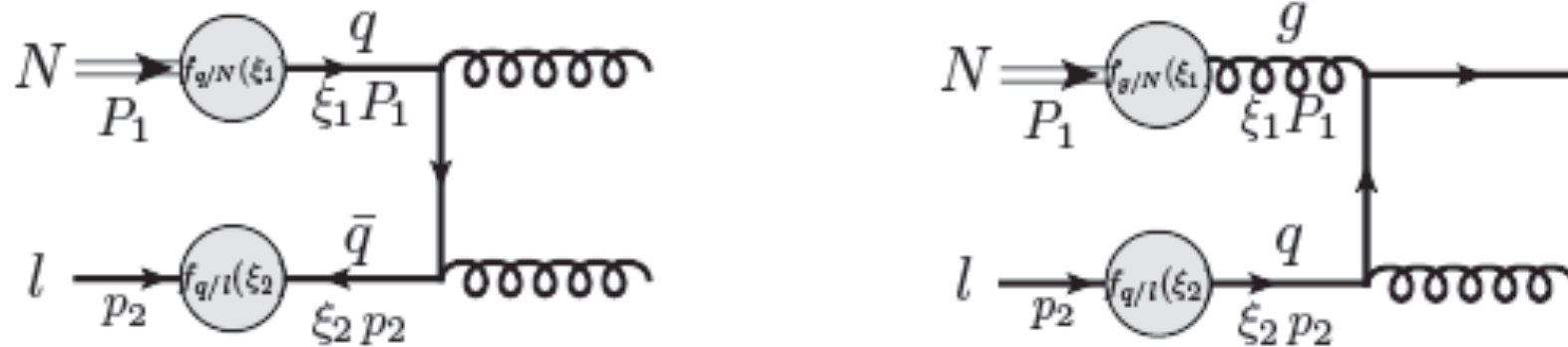


$$f_{\gamma/l}(\xi) = \frac{\alpha}{2\pi} P_{\gamma l}(\xi) \left[\ln \left(\frac{\mu^2}{\xi^2 m_l^2} \right) - 1 \right] + \mathcal{O}(\alpha^2)$$

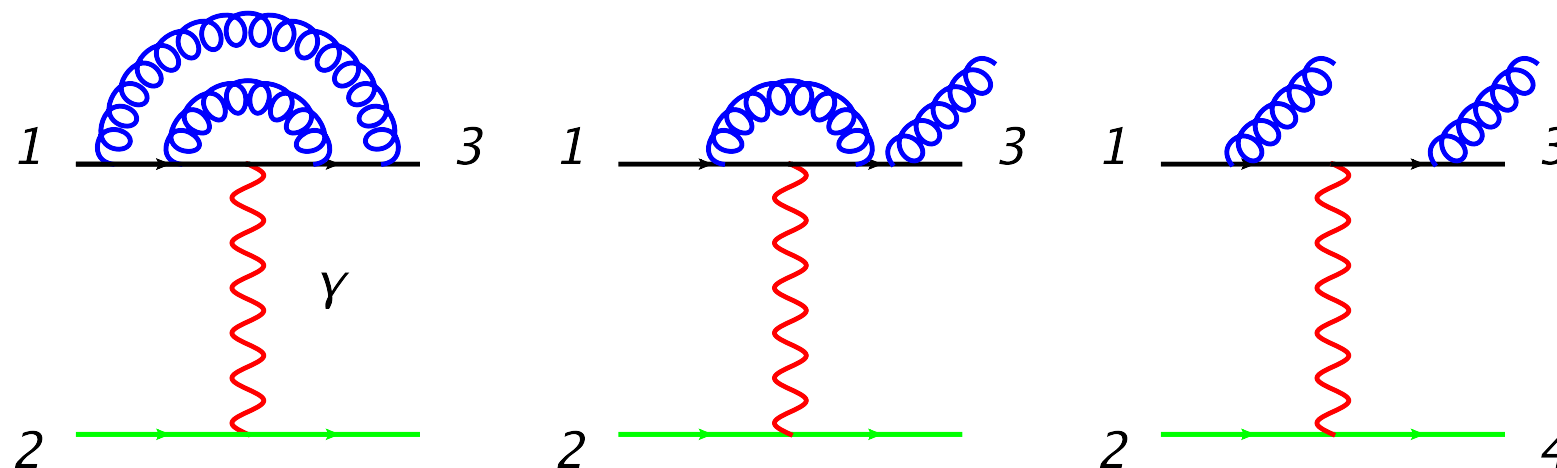
$$P_{\gamma l}(\xi) = \frac{1 + (1 - \xi)^2}{\xi}$$

NNLO $O(\alpha^2\alpha_s^2)$ corrections

- **New configuration:** incoming lepton can split into a quark, leading to parton-parton scattering channels. They first appear at this order, and are therefore effectively leading order in our treatment.



- Standard NLO corrections to quark-photon scattering
- Double-virtual, real-virtual, and double-real corrections to quark-lepton scattering

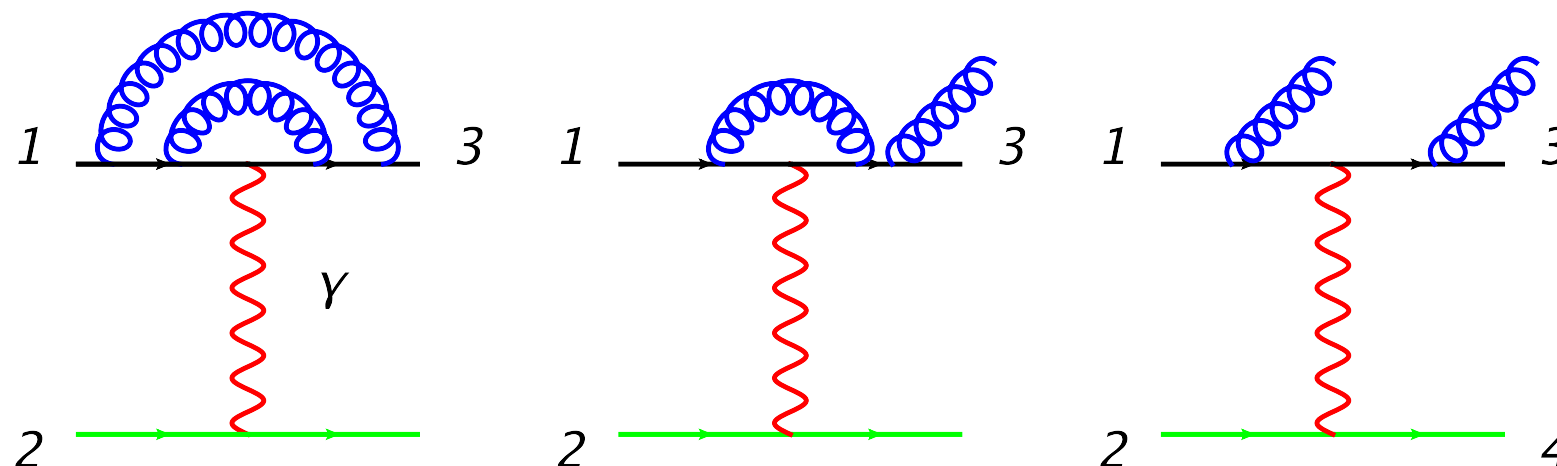


NNLO $O(\alpha^2\alpha_s^2)$ corrections

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All contributions separately divergent, with numerous singular configurations (triple-collinear, double-soft, soft+collinear, etc.)
How do we regularize and cancel to arrive at a finite result?

- Standard
- Double lepton scattering



NNLO subtraction

Enormous progress solving this problem for LHC physics!

- First complete predictions for LHC V+jet, Higgs+jet production at NNLO; partial results for di-jet production
- **N-jettiness subtraction**: (Boughezal, Focke, Liu, FP (2015); Gaunt, Stahlhofen, Tackmann, Walsh (2015))

$$\mathcal{T}_1 = \frac{2}{Q^2} \sum_i \min \{p_B \cdot q_i, p_J \cdot q_i\}$$

Stewart, Tackmann, Waalewijn (2010);
D. Kang, Lee, Stewart (2013)

p_B : beam axis

p_J : leading-jet axis

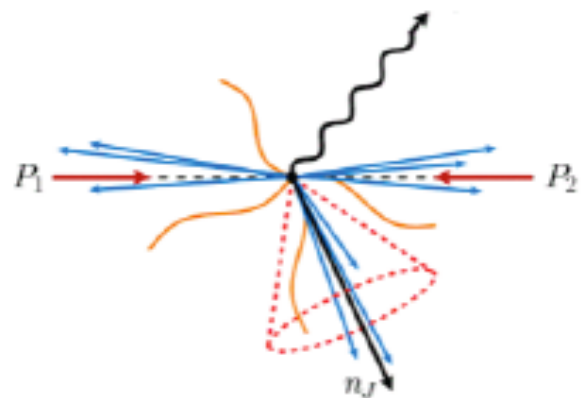
q_i : outgoing parton momenta

1 jet $\xleftarrow{\text{Small}} \mathcal{T}_1 \xrightarrow{\text{Large}}$ At least 2 jets

Contributions from

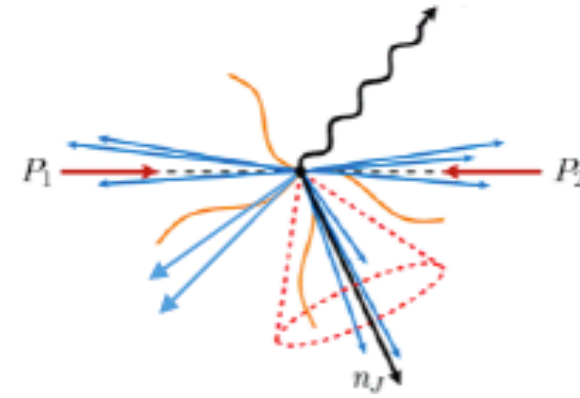
❖ Two-loop

❖ Soft and collinear radiation



Contributions with at least 2 hard jets

❖ NLO two-jet calculation. Use known results/tools



N-jettiness subtraction

- N-jettiness can be applied to obtain exact NNLO cross sections
- Introduce τ_N^{cut} that separates the $\tau_N=0$ doubly-unresolved limit of phase space from the single-unresolved and hard regions

$$\begin{aligned}\sigma_{NNLO} &= \int d\Phi_N |\mathcal{M}_N|^2 + \int d\Phi_{N+1} |\mathcal{M}_{N+1}|^2 \theta_N^< \\ &+ \int d\Phi_{N+2} |\mathcal{M}_{N+2}|^2 \theta_N^< + \int d\Phi_{N+1} |\mathcal{M}_{N+1}|^2 \theta_N^> \\ &+ \int d\Phi_{N+2} |\mathcal{M}_{N+2}|^2 \theta_N^> \\ &\equiv \sigma_{NNLO}(\mathcal{T}_N < \tau_N^{\text{cut}}) + \sigma_{NNLO}(\mathcal{T}_N > \tau_N^{\text{cut}})\end{aligned}$$

$$\theta_N^< = \theta(\tau_N^{\text{cut}} - \tau_N) \quad \text{and} \quad \theta_N^> = \theta(\tau_N - \tau_N^{\text{cut}})$$

N-jettiness subtraction

- For $\tau_N > \tau_N^{\text{cut}}$, at least one of the two additional radiations that appear at NNLO is resolved; this region of phase space contains the NLO correction to the N+1 jet process. **A solved problem!**
- For $\tau_N < \tau_N^{\text{cut}}$, both additional radiations are unresolved. A factorization theorem giving the all-orders result for small N-jettiness was derived
Stewart, Tackmann, Waalewijn 0910.0467

$$\frac{d\sigma}{d\mathcal{T}_1} = \int d\Phi_B \int dt_J dt_B dk_S \delta \left(\mathcal{T}_1 - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q} \right) \\ \times \sum_q J_q(t_J, \mu) S(k_S, \mu) H_q(\Phi_2, \mu) B_q(t_B, x, \mu) + \dots$$

H: describes hard radiation; in dim-reg, coincides with the 2-loop virtual corrections

B: describes radiation collinear to an initial-state beam

S: describes soft radiation

J: describes radiation collinear to a final-state jet

- The ellipses denote power corrections that become negligible for small τ_N^{cut}

Ingredients for the factorization theorem

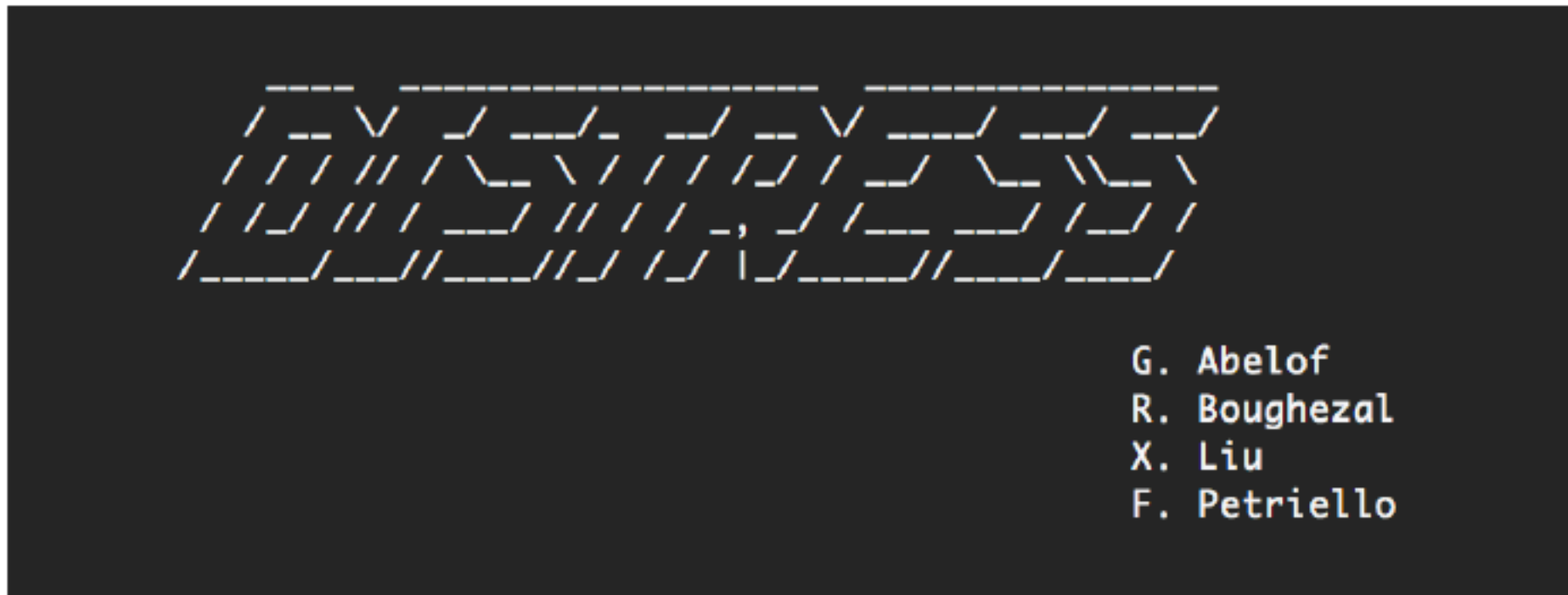
$$\frac{d\sigma}{d\mathcal{T}_1} = \int d\Phi_B \int dt_J dt_B dk_S \delta \left(\mathcal{T}_1 - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q} \right) \\ \times \sum_q J_q(t_J, \mu) S(k_S, \mu) H_q(\Phi_2, \mu) B_q(t_B, x, \mu) + \dots$$

- Expand this formula to $\mathcal{O}(\alpha_s^2)$, and turn off all resummation, to get the NNLO cross section below the cut. Need each of these separate functions to NNLO.
- The beam and jet functions depend only on the flavor of the parton (quark, gluon); the soft function depends only on the parton flavors and the external hard directions; the hard function is the only process-dependent piece.
- $H@NNLO$: Matsuura, van der Merck, van Nerven (1988)
- $B@NNLO$: Gaunt, Stahlhofen, Tackmann (2014)
- $S@NNLO$: Boughezal, Liu, FP (2015)
- $J@NNLO$: Becher, Neubert (2006); Becher, Bell (2011)

Within the past two years all ingredients have become available to apply this idea to jet production at the EIC!

DISTRESS

- **DISTRESS: DIS T**hrough a **R**obust **E**nabling **S**ubtraction **S**cheme



- Parton-level integrator for inclusive jet production in eN collisions
- Fully differential, allowing for arbitrary cuts on final-state jets/leptons
- Parallelized Monte Carlo integration
- Flexible framework allows for future extension to other processes (SIDIS, polarized collisions, ...)

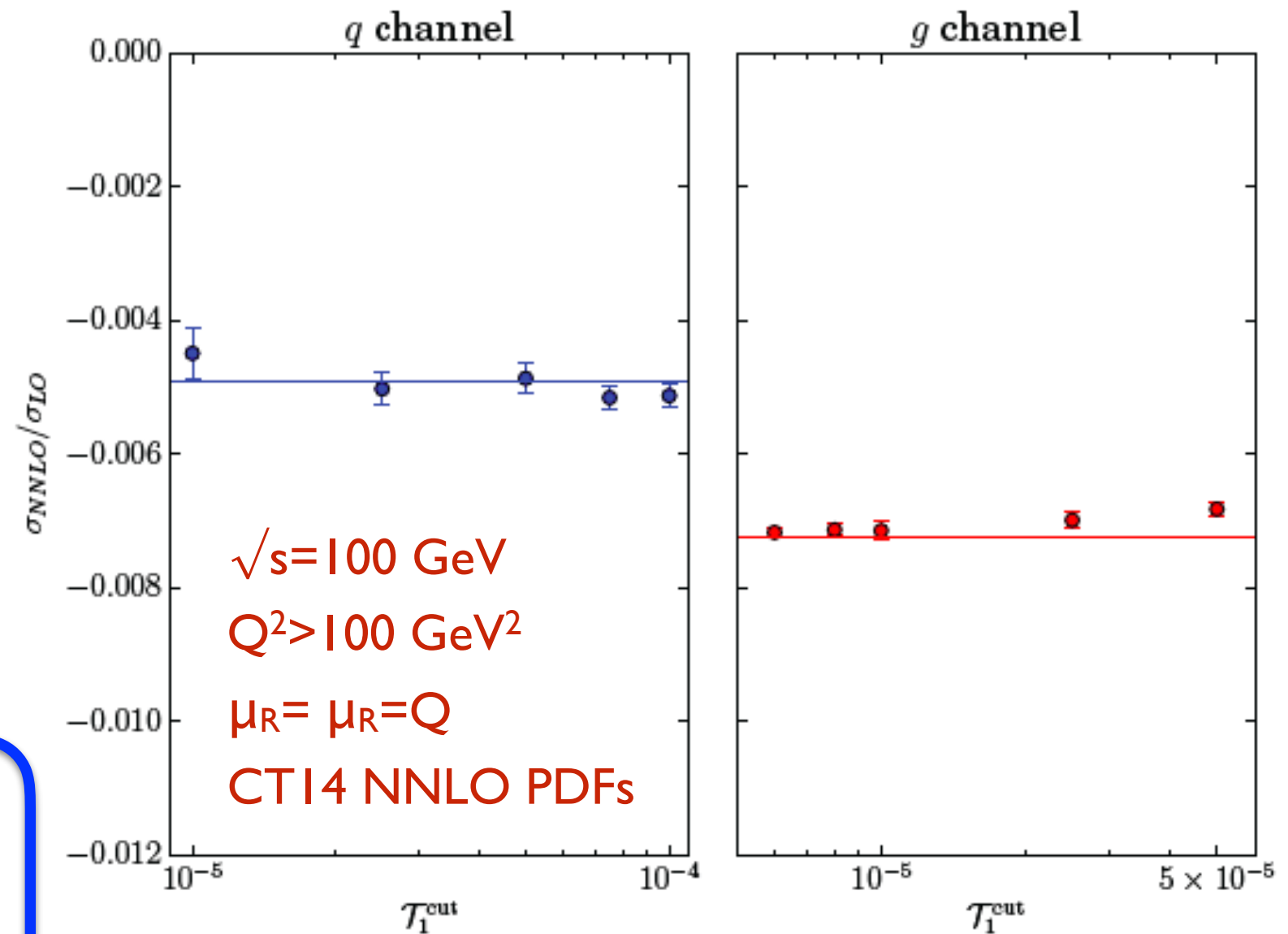
Validation

- We have two primary checks of our result at NNLO:

1. Independence of the full result from τ_1^{cut} ; also determines when power corrections are negligible
2. Upon integration over final-state radiation, must reproduce inclusive structure function

**Agreement with NNLO
structure function**

Zijlstra, van Neerven (1992);
Moch, Vermaseren (1999)



Numerics: setup and Q^2 distribution

- Study the predictions from DISTRESS for possible future EIC parameters:

$$\sqrt{s} = 100 \text{ GeV}$$

$$p_{T\text{jet}} > 5 \text{ GeV}$$

$$|\eta_{\text{jet}}| < 2.0$$

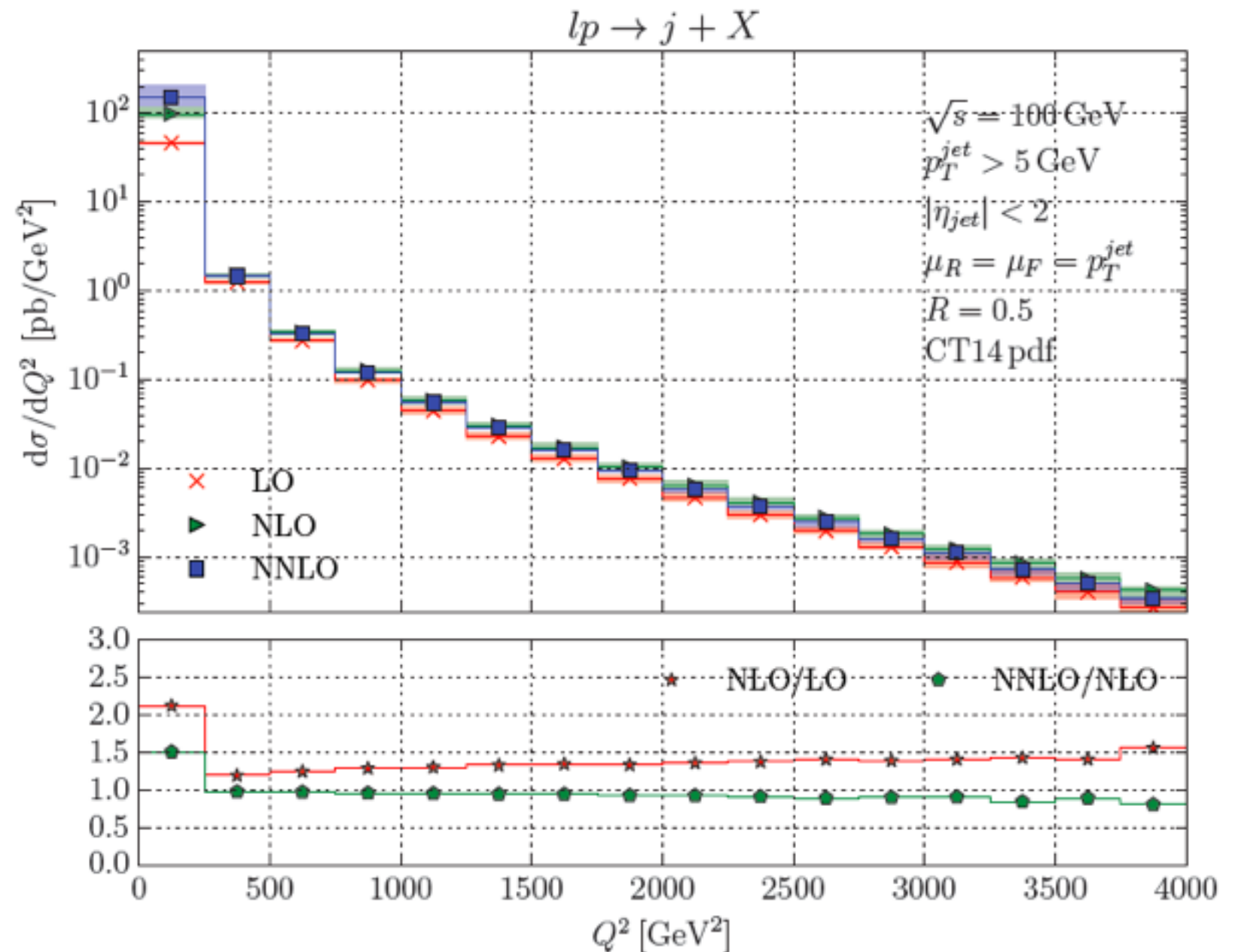
$$\text{Anti-}k_T, R=0.5$$

$$\mu_R = \mu_F = p_{T\text{jet}}$$

$$\alpha = 1/137.036$$

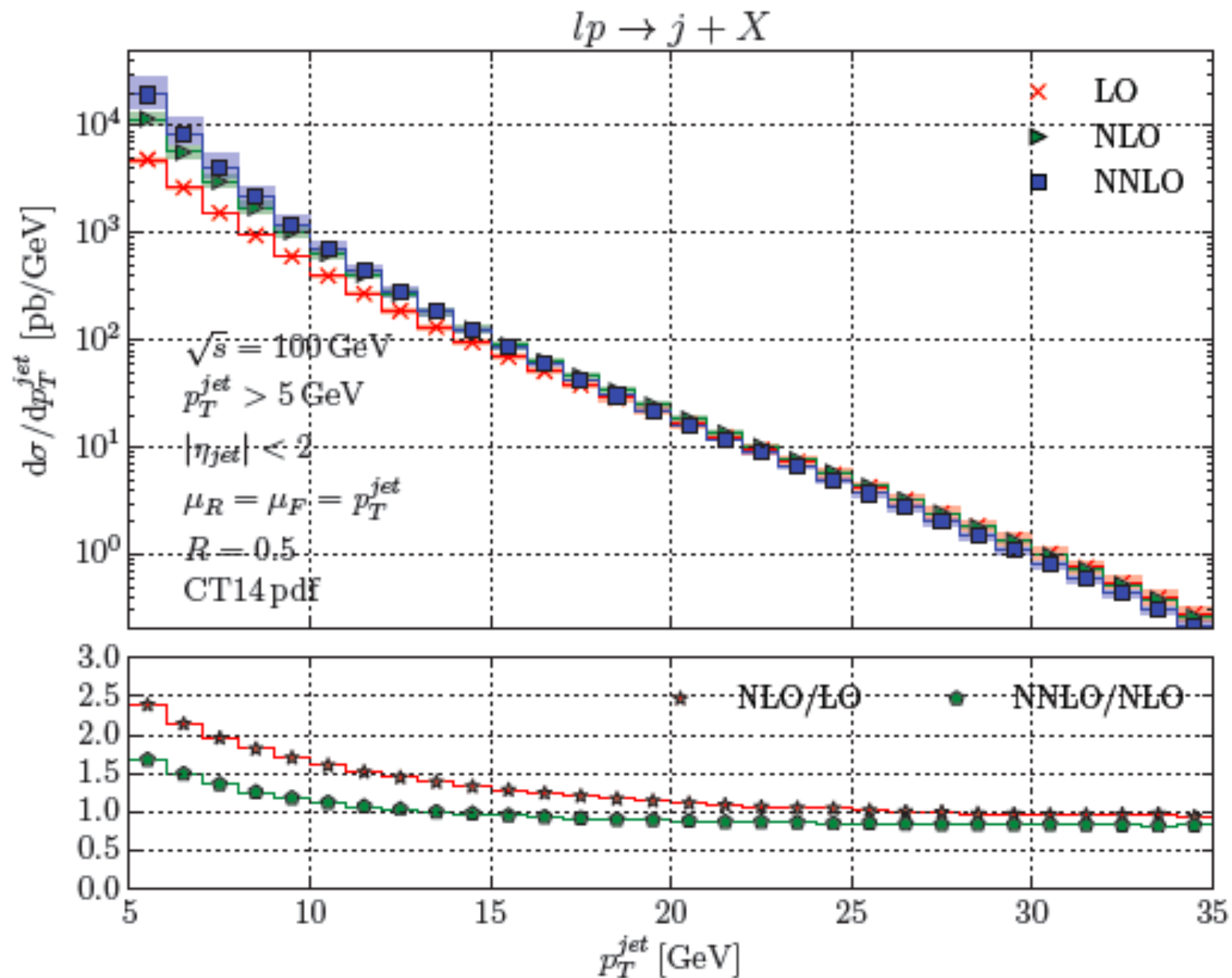
$$m_e = 0.511 \text{ MeV}$$

$$\text{CT14 PDFs}$$



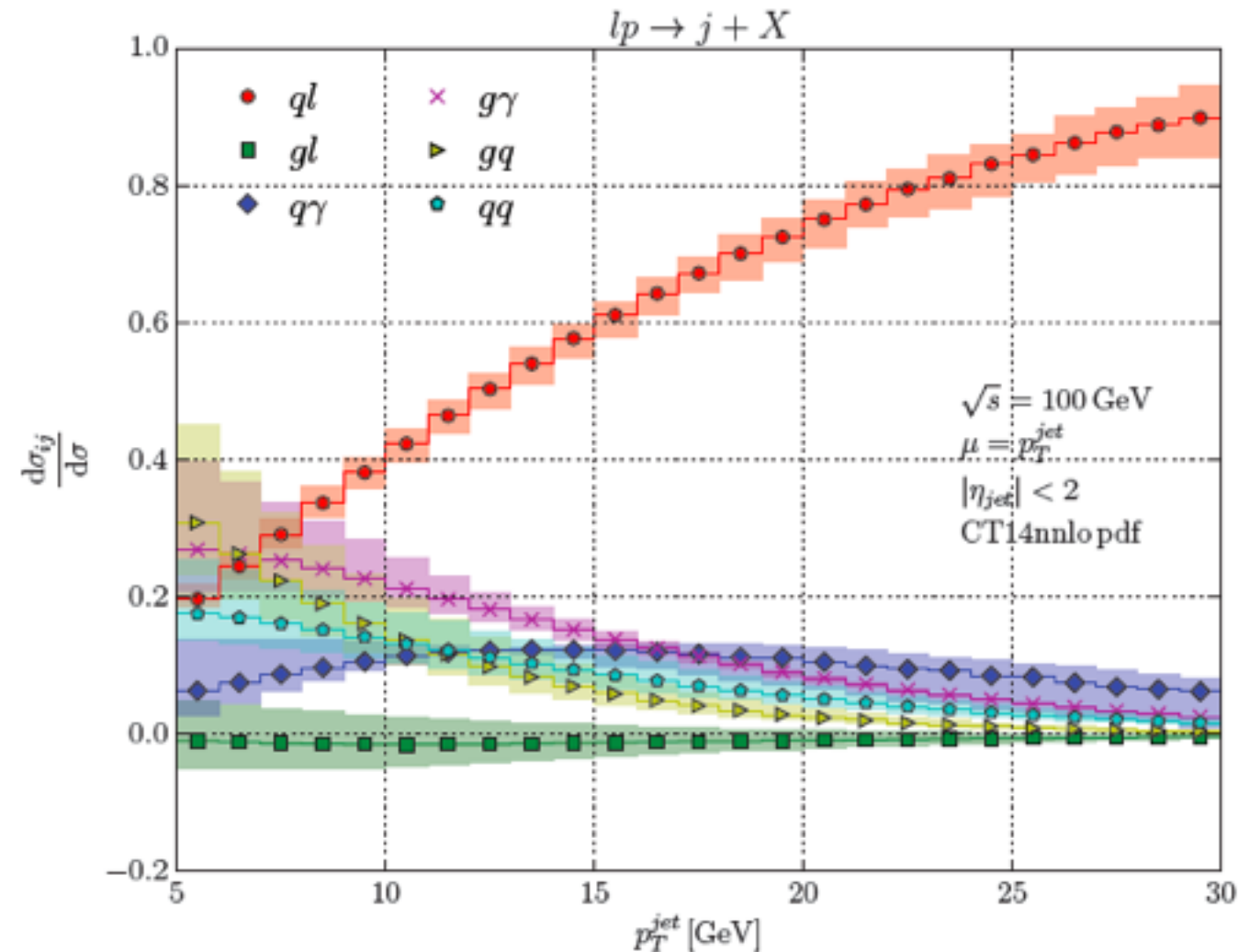
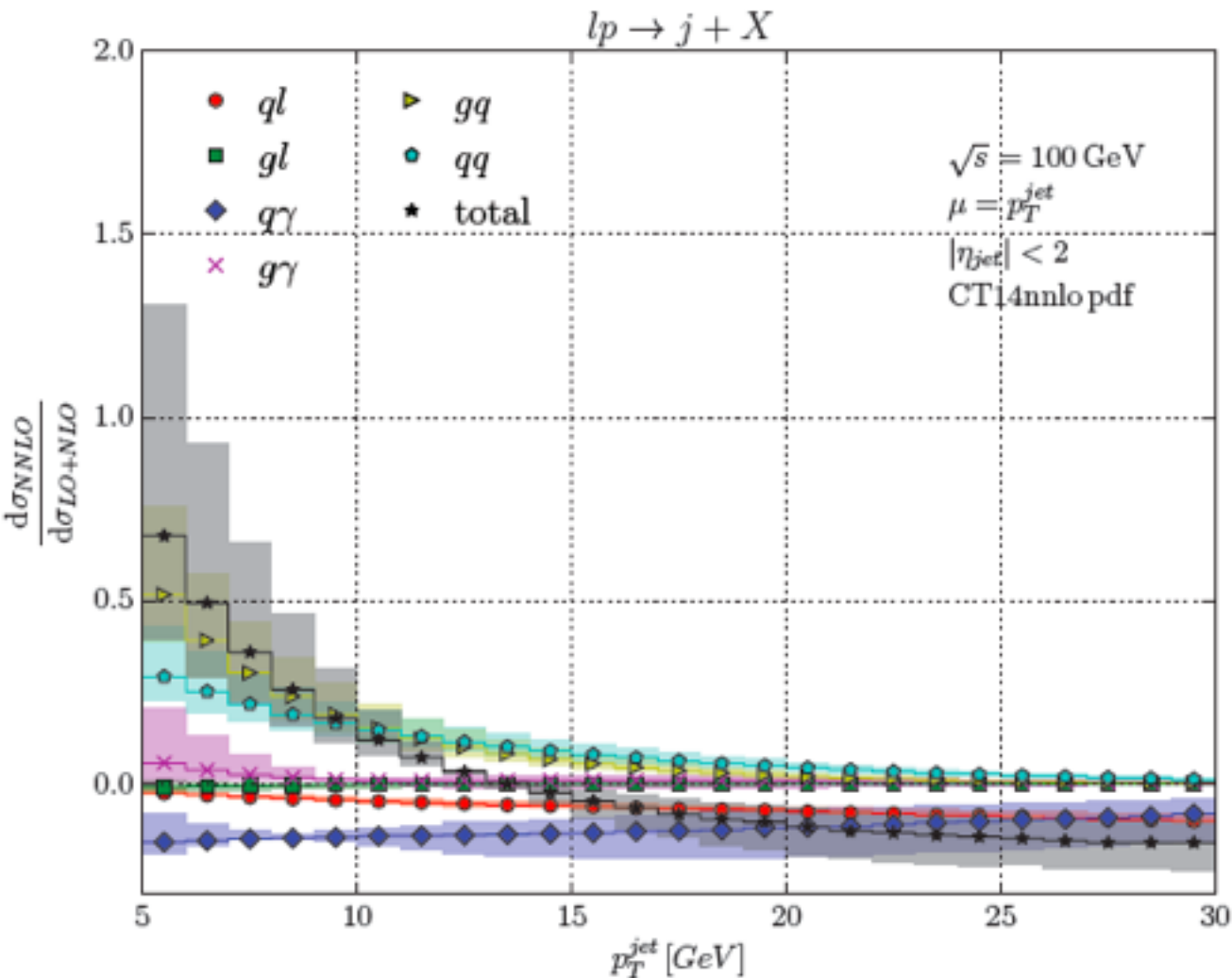
Large corrections at low Q^2 (photon-initiated processes)

Numerics: $p_{T,\text{jet}}$ distribution



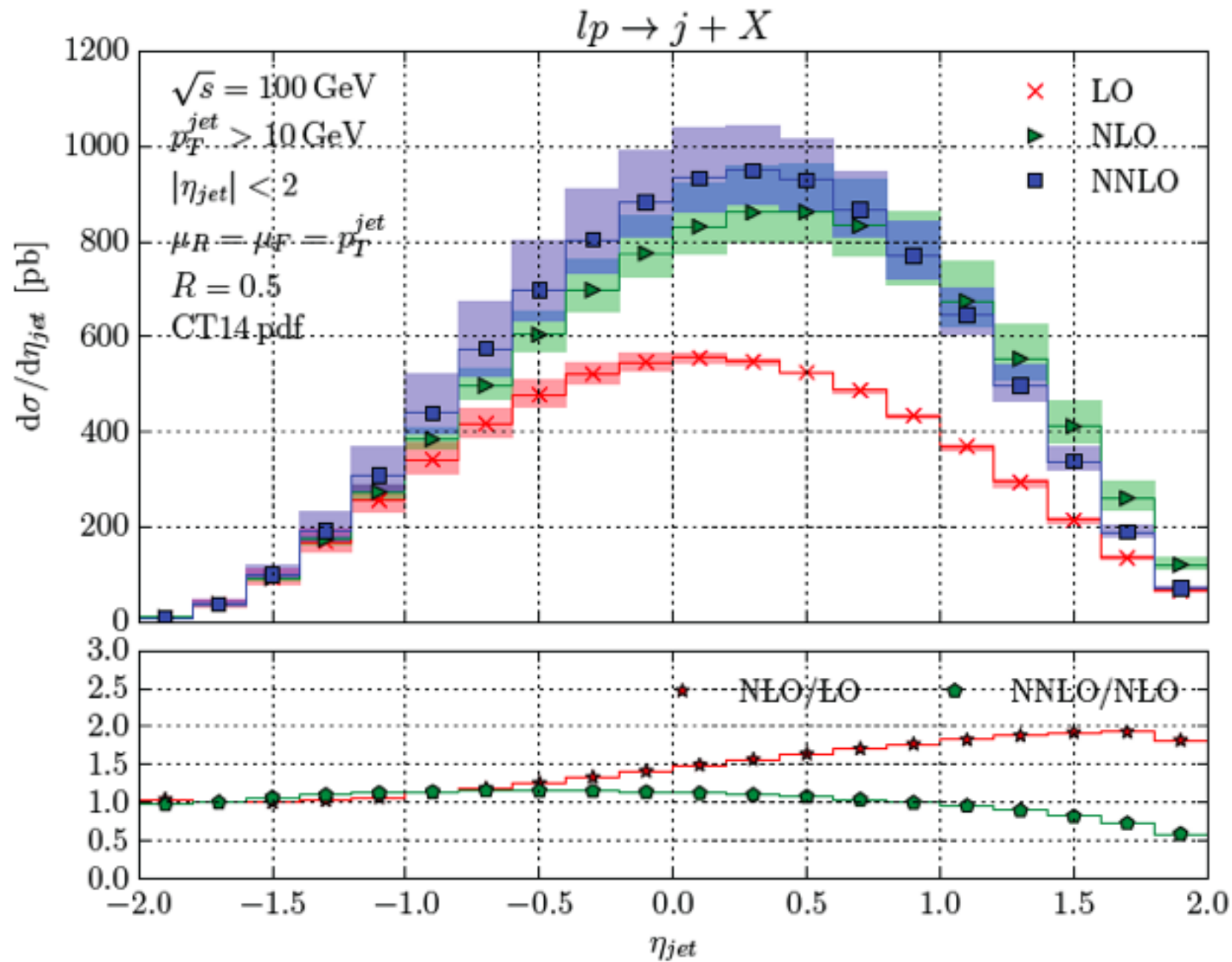
- NNLO large and positive for $p_{T,\text{jet}} < 10$ GeV; near unity for large momenta
- Scale dependence *increases* at NNLO for $p_{T,\text{jet}} < 10$ GeV

$p_{T,\text{jet}}$ distribution: partonic channels



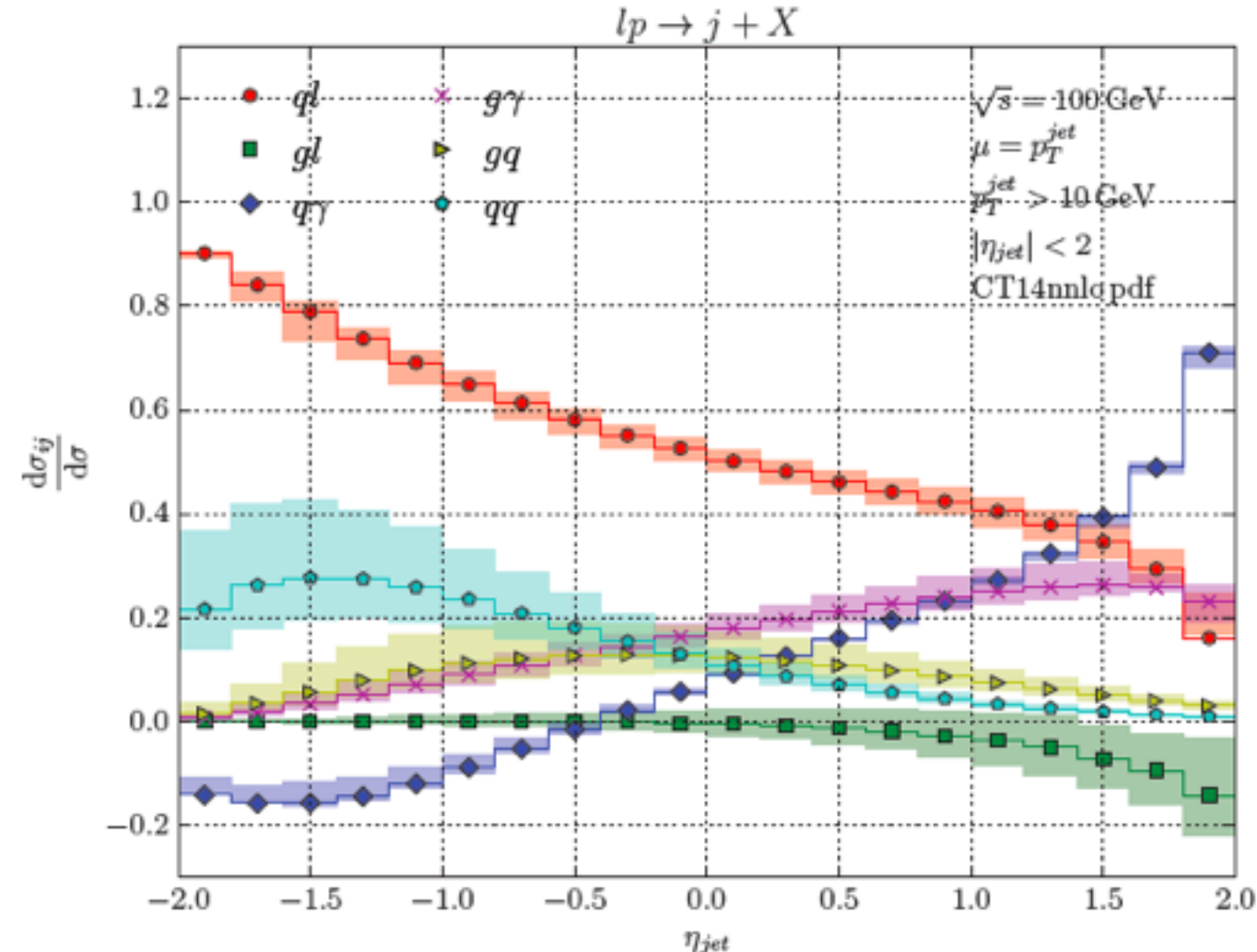
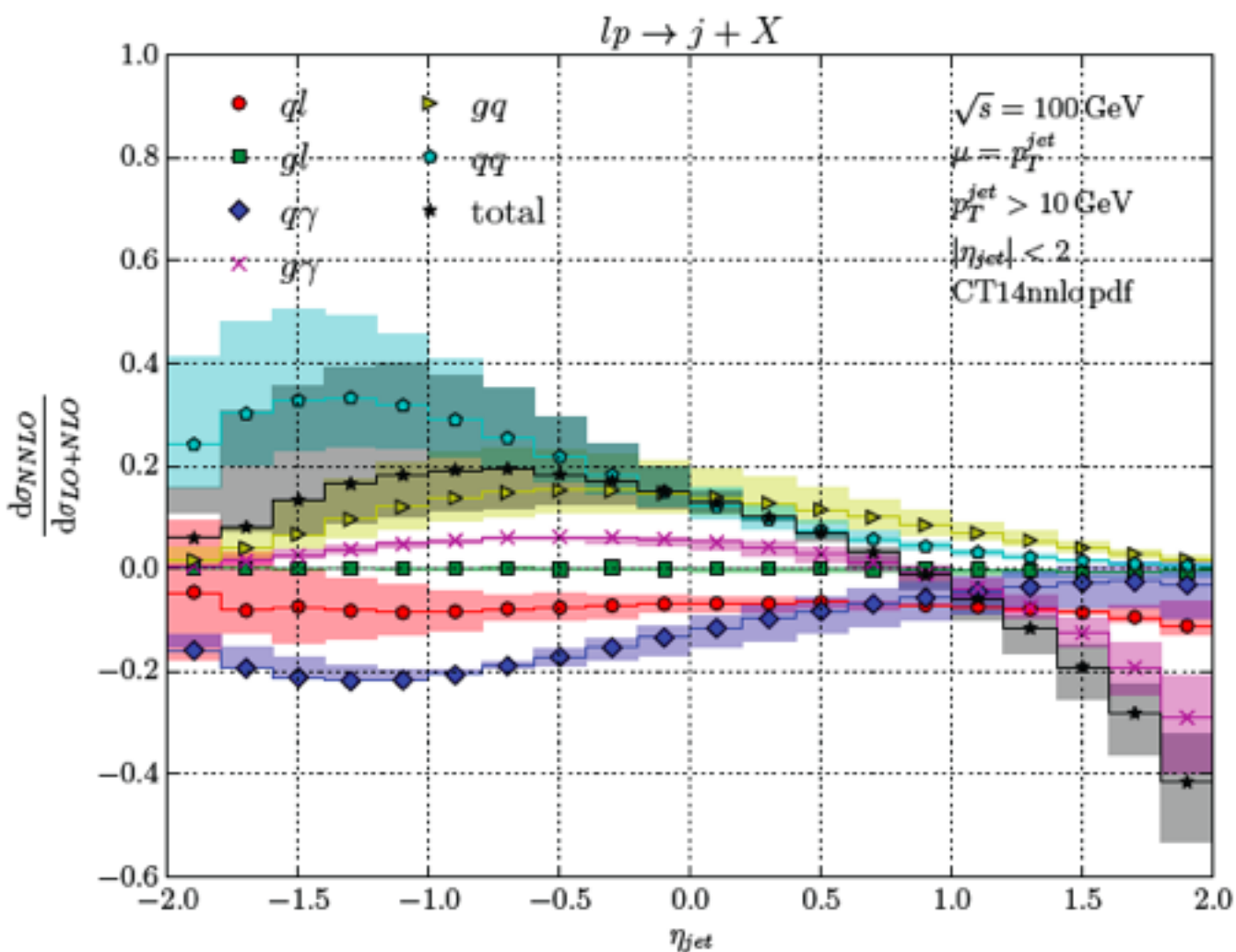
- qq and qg dominate the NNLO correction for low $p_{T,\text{jet}}$
- These channels begin at $\mathcal{O}(\alpha^2\alpha_s^2)$, are effectively leading-order in this result, and drive the increased scale dependence at NNLO
- ql channel dominates for high $p_{T,\text{jet}}$
- No single channel furnishes a good approximation to the full result

η_{jet} distribution



- NNLO corrections small for $\eta_{\text{jet}} < 1$, but increase as $\eta_{\text{jet}} \rightarrow 2$
- Scale dependence *increases* at NNLO for $\eta_{\text{jet}} < 0$

η_{jet} distribution: partonic channels



- qq channel drives the large scale uncertainty for $\eta_{\text{jet}} < 0$; it begins at $\mathcal{O}(\alpha^2\alpha_s^2)$, and is effectively leading-order in this result,
- ql channel dominates for low η_{jet} ; $q\gamma$ channel dominates at high η_{jet}
- No single channel furnishes a good approximation to the full result

Conclusions

- We have presented a calculation of the full $O(\alpha^2\alpha_s^2)$ corrections to inclusive jet production at a future EIC
- Our calculation allows for arbitrary final state cuts as is implemented in the pardon-level program **DISTRESS**
- The magnitude of the corrections indicate that higher-order corrections will play an important role in the future EIC program
- Many additional EIC applications are possible using the techniques developed here; **stay tuned!**